D 43225 (Pages : 3) Name..................................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS–UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*
Each question carries 1 *mark.*

1. Write an example for a sequence which has no upper bound.
2. Find the domain of the function $w = xy \ln z$.
3. Define the level surface of a function f.
4. State two path test for non-existence of limit.
5. If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$.
6. $\frac{d}{dx} \sinh x =$ ————.
7. Write tanh x in terms of exponential function.
8. Find $\lim_{n \to \infty} \sqrt[n]{n}$.
9. $\int \cosh 2x =$ ————.
10. Find $\lim_{(x,y) \to (1,1)} \frac{x^2 + 2y}{3x - 2}$.
11. Find $\frac{\partial}{\partial x} \sin 2xy$.
12. Define conditional convergence of a series.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Investigate the convergence of $\int_0^{\infty} e^{-x^2} dx$.
14. Show that $\lim_{n \to \infty} k = k$, where k is a constant.

Turn over

15. Find $\lim_{n \to \infty} \frac{\cos n}{n}$.

16. Find $\lim_{(x,y) \to (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

17. Show that the function $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.

18. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

19. Use chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. What is the derivative's value at $t = \pi/2$?

20. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

21. Show that if u is a differentiable function of x whose values are greater than 1, then $\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}$.

22. Graph the sets of points whose co-ordinates satisfies the condition $2\pi/3 \leq \theta \leq 5\pi/6$ (no restriction on r).

23. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.

24. Find the directrix of the parabola $r = \frac{25}{10 + 10\cos\theta}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Compare $\int_1^\infty \frac{dx}{x^2}$ and $\int_1^\infty \frac{dx}{1 + x^2}$ with limit comparison test.

26. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent or divergent.

27. Find the linearization of the function $f(x, y) = x^2 + y^2 + 1$ at $(0, 0)$.

28. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s$ and $y = r + s$.

29. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

30. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = e^{3x+4y} \cos 5z$.

31. Find the Maclaurin series for the function $f(x) = xe^x$.

32. Does series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ convergent.

33. Find the surface area generated by revolving the curves $x = t + \sqrt{2}$, $y = \frac{t^2}{2} + \sqrt{2}t$, $-\sqrt{2} \le t \le \sqrt{2}$ about y-axis.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

35. Find the points of intersection of $r^2 = 4\cos\theta$ and $r = 1 - \cos\theta$.

36. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the functions's local and absolute extrema values.

(2 × 10 = 20 marks)

C 62623 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2019

(CUCBCSS–UG)

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Types)

Answer all **twelve** *questions.*

1. Define a sequence.

2. Fill in the blanks : $\frac{d}{dx}\cosh^3(3x) =$ ————————.

3. For what values of real numbers x, does the series $\sum_{n=1}^{\infty} \sin^n x$ converge ?

4. Fill in the blanks : The polar equation of the circle with centre origin and radius a is ————————.

5. Find the n^{th} term of the sequence 2, – 2, 2, – 2 ————————.

6. Fill in the blanks : If $f(x,y) = 1 - \sinh(1 - xy)$, then $f_x(1,1) =$ ————————.

7. Fill in the blanks : If f is continuous on $[a,b)$, then $\lim_{c \to b} \int_a^c f(t)\,dt =$ ————————.

8. Write explicitly the ratio test for the convergence of the series $\sum_{n=0}^{\infty} a_n$.

9. State alternating series test of Leibniz.

10. Define $\frac{\partial}{\partial x} f(x,y)$ using limit.

11. The power series $\sum_{n=0}^{\infty} a_n (x-a)^n$ always converges to a_0 when $x =$ ————————.

12. What do you mean by linearization of a function in two variables at a point.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Types)

Answer any **nine** *questions.*

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.

14. Test the convergence of the integral $\int_0^{1/2} \frac{1}{1-2x} dx$.

15. State the non-decreasing sequence theorem.

16. Describe the level surface of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2-1}$.

17. Graph the sets of points whose polar co-ordinates satisfy the condition $0 \le r \le 2$.

18. Evaluate $\int_0^1 \frac{3dx}{\sqrt{4+9x^2}}$.

19. Find tanh x, if $\cosh x = \frac{17}{15}, x > 0$.

20. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ if $f(x,y) = \log\sqrt{x^2+y^2}$.

21. Find a cylindrical co-ordinate equation for the surface $x^2 + (y-3)^2 = 9$.

22. Find $\frac{\partial z}{\partial r}$ if $z = x + 2y$, $x = \frac{r}{s}$ and $y = 2rs$.

23. Find $\lim_{n \to \infty} \frac{n}{2n+1}$.

24. Write the Maclaurin series for sin x.

(9 × 2 = 18 marks)

Part C (Short Essay Types)

Answer any **six** *questions.*

25. Find the length of the curve $y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$.

26. Find the limit of the function $f(x,y) = \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ as (x,y) tends to $(0, 0)$.

Replace the polar equation $r = \dfrac{4}{2\cos\theta - \sin\theta}$ by equivalent Cartesian equation and the draw the graph in Cartesian form.

28. Find a power series for log $(1 + x)$ and find the radius of convergence of that series.

29. Show that $\tanh^{-1} x = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right)$.

30. Find the volume of the solid of revolution when the region between the parabola $x = y^2 + 1$ and the line $x = 3$ is revolved about the line $x = 3$.

31. Find the sum of the series $\sum_{n=1}^{\infty} \dfrac{2^n - 1}{4^n}$.

32. Find the radius and interval of convergence of the series : $\sum_{n=0}^{\infty} (-1)^n (2x-1)^n$.

33. Evaluate : $\int \dfrac{\cosh^4 \sqrt{x}}{\sqrt{x}}\, dx$.

(6 × 5 = 30 marks)

Part D (Essay Types)

Answer any **two** *questions.*

34. Show that the function $f(x,y) = \dfrac{2xy}{x^2 + y^2}$ when $(x,y) \neq (0,0)$ and 0, otherwise is continuous everywhere except at the origin.

35. (a) Find the linearization of the function $f(x,y) = x^2 - xy + y^2/2 + 3$ at (3, 2).

    (b) Find the area of the region enclosed by the cardioid : $r = 2(1 + \cos\theta)$.

36. Find the area of the surface generated by revolving the curve $y = x^3/9, 0 \le x \le 2$ about the x-axis.

(2 × 10 = 20 marks)